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Preliminary Study to Detect Match-Fixing: Benford's Law in Badminton Rally Data

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Abstract

To investigate the possibility of detecting match-fixing, this study aims to verify whether the number of rallies in universal and anomalous badminton matches follows Benford's law. The study counted the number of rallies in 685 international badminton tournaments selected through purposive sampling and analyzed the first significant digits of the rally data through the χ^2 test and intra class correlation coefficient. Of these, two well-known fixed badminton matches were found to be anomalous, and hence, the finding is that the number of badminton rallies in universal matches follows Benford's law while the anomalous matches do not do so.

Keywords: Benford's law, match-fixing, detecting anomalous matches, fixed badminton matches

1. Introduction

It may appear that the frequency of individual numbers between 1 and 9 appearing as the first digits of the infinite natural numbers used in real life, such as various economic indicators, geo-statistical data, and crime rates, is the same for each of these digits, at 11.1%. However, this is not true, as the rate forms a distribution with a certain rule. That is, the rate of using 1 as the first digit of a number is disproportionate to the rate of using the rest of the numbers, and the lower the digit, the higher its frequency (Newcomb, 1881).

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Benford's law (Benford, 1938) refers to the phenomenon in which as the first digit of a number increases from 1 to 9, the occurrence rate of this number reduces, as the rate of the first digit being 1 is about 30.10%, that of 2 is 17.61%, and that of 9 is about 4.58% (Miller, Berger, & Hill, 2010). Benford (1938) proved the fact that the numbers starting with 1 are more specifically distributed among the numbers that are used in every day such as the area of the lake, demographics, mortality statistics, and numbers appearing in newspapers, and these numbers follow a unique rule, called Benford's law, which is illustrated below.

$$P_D = \log10\ (1+1/D) \qquad \text{(Formula 1)}$$

P_D represents the occurrence probability of the first significant digit (FSD), D (D=1, 2, ..., 9) of a number. For 50 years after Benford (1938) reported this law, Benford's law was regarded merely as a mathematical phenomenon, with its applicability in real life being low. However, as vast amounts of data are applied to Benford's law accordingly with the development of computer processing technology, interesting research results that followed this law arose. Regarding the characteristic of the probability distribution that appears in data in which Benford's law is produced spontaneously, the fact that artificially produced data does not follow Benford's law can be speculated. It is because the first digit of man-made artificial numbers is assigned uniformly from 1 to 9. Based on this assumption, Benford's law is applied for, say, detecting fraudulent activities that manipulates corporate accounts (Hill, 1998; Miller, Berger, & Hill, 2010) and for verifying the veracity of survey data (Judge & Schechter, 2009).

By using Benford's law, Nigrini (1992) introduced a basic idea for detecting tax evaders and Hill (1998) and Nigrini & Mittermaier (1997) presented a model that detects manipulation of accounting. Varian (1972) argued that although conformity with Benford's law does not necessarily ensure the veracity of data, data veracity may be questioned in case of inconformity. The fact that the Greek national accounts manipulation was identified by Benford's law in 2011 (Rauch, Göttsche, Brähler, & Engel, 2011) is well-known. The present study aims to investigate whether the data on sports tournaments such as badminton rallies would conform to Benford's law. Given that the accounting data generated, which involves corporate competition, conforms to Benford's law, there is no reason that sports tournament data that are generated as a result of competition between players would not follow Benford's law.

Sports data generated as a result of competitive matches are natural numbers like the real-life sets of numbers that follow Benford's law such as the height of mountains and buildings and the width of rivers. Therefore, determining whether particular sports data conforms to Benford's law can be applied to detect unnaturally performed anomalous matches (e.g., match-fixing). For instance, if data on length of badminton rallies that is collected in a particular badminton match does not follow Benford's law despite the fact that data on length of badminton rallies conforms to this law, it may imply that the competition has not been carried out as per naturally occurring perfect competition.

Generally, numbers with less than 4 digits, numbers with fixed ranges, such as adult height and IQ, artificially determined numbers such as $1.99, and rounded numbers are known to not follow Benford's law (Diekmann & Jann, 2010; Günnel & Tödter, 2009). Although the number of badminton rallies generally has two digits, it can be infinite starting from 1 since the range is not determined theoretically. Further, the number of rallies can be considered as a variable for application of Benford's law among the various badminton records since it cannot be determined by players artificially as well as the integer numbers are used. The objective of the present study is to verify whether badminton rally data in universal matches follows Benford's law. In addition, this study is to investigate the possibility of detecting match-fixing using Benford's law by verifying whether cases of anomalous matches that are well-known match-fixing games follow Benford's law.

2. Methods

2.1. Data Sources

2.1.1. Universal badminton matches

The data used in this study are all of 685 badminton matches that are collected from purposive sampling of international tournaments from 2012 to 2016 for the analysis of Korea's national badminton team. This comprises 64 international badminton tournaments, including Denmark Open, India Open, Japan Open, and Korea Open. The 685 games were played by athletes who ranked in the world's top 20. There were 127 games of men's singles, 176 games of women's singles, 184 games of men's doubles, 102 games of women's doubles, and 96 games of mixed doubles.

In the 685 badminton match videos, the number of rallies was counted by each point scored such that service was recorded as rally 1, receiver as rally 2, third ball play as rally 3, and so on. The number of rallies in the data ranges from 1 to 185, with an average of 10.20 and a standard deviation of 8.68. The total number of valid cases that are collected from the entire 685 games and used in the analysis is 57,404, and it is assumed that the data represent the characteristics of badminton, as the badminton rally data were collected from a wide range of tournaments of the world's best players.

2.1.2. Anomalous badminton matches

In this study, the rally data of two well-known fixed badminton matches as anomalous cases were collected. One case was the women's doubles preliminary round at the 2012 London Olympic Games in which South Korean and Chinese pairs competing with each other intentionally lost the match in order to obtain higher seed so as to avoid playing with the 1st ranking team subsequently (Chappelet, 2015). The Badminton World Federation (BWF) determined this match as a fixed game and disqualified the players. The other case was one of the matches in 2008 All England Championship in which the then globally 1st ranked Chinese player who had already been qualified for the Olympics intentionally lost against another Chinese player who was in need of being qualified for the Olympics. Consequently, the two Chinese players were able to participate together in the Olympics. BWF flagged this match for potential match-fixing (Badminton information, 2016). A total of 157 valid cases of rallies were collected from those two anomalous matches.

2.1.3. Benford's expected value vs. badminton rally data

Benford's law is used for verifying the normality of the numbers of a data set. That is, occurrence rate of 1 as the first digit of the number among the numbers naturally occurring in real life is 30.1%, which is about six times higher than that of 9, which is 4.58% (Benford, 1938; Nigrini, 1999). Based on this, if the difference in the rate of the numbers that occur in the real world is significant compared to Benford's law, the veracity of the number may be questioned. However, not all data conform to this law. Nigrini (1999) claimed that sample data should have the following conditions for the application of Benford's law. Data should be numbers, and numbers should be generated for the same purpose, spontaneous and free from restriction of the range.

Given that the badminton rally data, which are sought to be applied in this study, are recorded in numbers and collected for the actions of players with the intention of competing; the size of the number of rallies is not limited theoretically; and the number of rallies cannot be adjusted artificially, the data conform to Benford's law conditions presented by Nigrini (1999).

This then leads to the question whether the case in which the number of badminton rallies is not spontaneous but manipulated (two anomalous matches selected in this study) conforms to Benford's law. Therefore, from an integrated perspective of men's singles, women's singles, men's doubles, women's doubles, and mixed doubles, in which the world's top 20 players participated, the study examined the conformity to Benford's law by comparing this law's reference values with the expected value of Benford's law (Table 1). This was done by classifying two anomalous badminton matches, a well-known case of match-fixing or intentionally defeated games, by set.

Table 1: Expected value of Benford's law

	First digit								
	1	2	3	4	5	6	7	8	9
Expected value	30.10	17.61	12.49	9.69	7.92	6.69	5.80	5.12	4.58

2.2. Statistical Test

Only the FSD was analyzed in this study, since Benford's law focuses on verifying the distribution of FSD and the number of badminton rallies is typically of two or fewer digits. The present study calculated the observed rate of badminton rally data from 1 to 9, which was collected from 685 matches and compared with Benford's expected rate. Moreover, the game that was the 2012 London Olympics deliberately defeating match was included for the analysis to detect match fixing as a preliminary study

For statistical verification for fitting between badminton rallies distribution and Benford's distribution, χ^2 test value and intra class correlation coefficient (ICC) were calculated. χ^2 test, as statistics that traditionally indicates the difference between observed frequency and expected frequency, is known as goodness-of-fit index.

The degrees of freedom of the χ^2 test are 8 and the formula is shown in Formula 2. ICC is generally an index indicating the conformity degree among two or more measured values (Formula 3). If the ICC value is greater than .90, observed value of badminton rallies can be regarded to be very consistent with the expected distribution of Benford (Portney & Watkins, 2000).

$$\chi^2 = \sum_{i=1}^{9} \frac{(o_i - e_i)^2}{e_i}$$ (Formula 2)

In Formula 2, o_i represents the expected rate of observed value and e_i is the expected rate of Benford's law. Baseline for hypothesis testing Benford's Law, α was set to .05 and at 8 degrees of freedom, and the χ^2 test value is 15.51.

$$ICC = \frac{\sigma_b^2}{\sigma_b^2 + \sigma_w^2}$$ (Formula 3)

σ_b^2 is the variance between couples and σ_w^2 is variance within couples. ICC is the rate of variance between couples among total variance of the two measurements (conformity degree).

3. Results

3.1. Universal Badminton Rally Data

Figure 1 graphically illustrates Benford's expected distribution and the distribution of the first digits of numbers of badminton event-specific rallies. The distribution of the first digit of numbers that targeted 57,404 badminton rallies shows that the number 1 has a ratio of 30% in all events including men's singles, women's singles, men's doubles, women's doubles, and mixed doubles, and is in line with Benford's law. All the observed rates in badminton rally data are perfectly identical to Benford's law except for the fact that the observed value of the number 2 is slightly low compared to Benford's distribution in the distribution of the first digit of badminton rallies number.

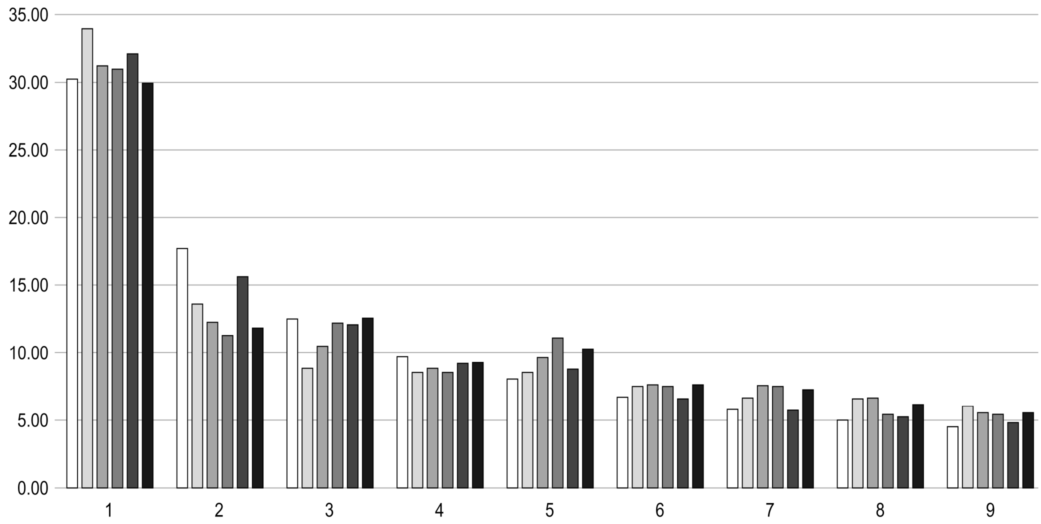

Figure 1: First-digit distribution in universal badminton matches.

Note) from the left side, Benford's expected value, men's singles, women's singles, men's doubles, women's doubles, and mixed doubles are presented in order. Observed values of first digit number of badminton rallies are as follows. In men's singles, 34% for 1, 14% for 2, 9% for 3, 9% for 4, 8% for 5, 7% for 6, 7% for 7, 7% for 8, 6% for 9; in women's singles, 31% for 1, 12% for 2, 10% for 3, 9% for 4, 10% for 5, 8% for 6, 8% for 7, 7% for 8, 6% for 9; in men's doubles, 31% for 1, 11% for 2, 12% for 3, 9% for 4, 11% for 5, 7% for 6, 7% for 7, 5% for 8, 5% for 9; in women's doubles, 32% for 1, 16% for 2, 12% for 3, 9% for 4, 9% for 5, 7% for 6, 6% for 7, 5% for 8, 5% for 9; in mixed doubles, 30% for 1, 12% for 2, 12% for 3, 9% for 4, 10% for 5, 8% for 6, 7% for 7, 6% for 8, 6% for 9

Table 2: Statistical verification of Benford's law in universal matches

Source	rallies			χ^2	$\underline{p}$	*ICC*[b]
	n	mean	max[a]			
Men's singles (n=127)	10821	11.64	86	3.68	.884	.961
Women's singles (n=176)	14517	9.26	56	3.79	.875	.962
Men's doubles (n=184)	15776	9.37	107	4.33	.825	.953
Women's doubles (n=102)	8166	12.90	185	0.47	.999	.993
Mixed doubles (n=96)	8124	8.85	108	3.45	.903	.960
Total (n=685)	57404	10.20	185	2.78	.947	.970

Max represents the maximum number of rallies. ICC denotes the intraclass correlation coefficient

Statistical verification was conducted to explore whether the distribution of the first digit of badminton event-specific rallies number conforms to Benford's law (Table 2). Given men's singles, (χ^2=3.68, p=.884, ICC=.961), women's singles (χ^2=3.79, p=.875, ICC=.962), men's doubles (χ^2=4.33, p=.825, ICC=.953), women's doubles (χ^2=0.47, p=.999, ICC=.993), and mixed doubles (χ^2=3.45, p=.903, ICC=.960), the distribution of the first digit of badminton rallies number was found to be in line with Benford's law. The entire 57,404 rallies of the 685 badminton tournaments were also found to conform to Benford's law with χ^2=2.78, p=.947, ICC=.970.

3.2. Anomalous Badminton Rally Data

In women's doubles during the 2012 London Olympics, Korean and Chinese players deliberately fixed the match, leading to several involved players and leaders receiving disciplinary measures.

The present study conducted an analysis of whether the distribution of the first digit of rallies number follows Benford's law in the well-known badminton fixed match. For the first and second sets, the distribution of the number of rallies was significantly far off from Benford's expected distribution (Table 3 and Figure 2). In the rally number of the first set, the frequency of the number 1 was 20% than Benford's expected distribution, while the numbers 5, 6, 7, 8 and 9 never occurred (χ^2=56.3, p=.001, ICC=.76). Although the Korean and Chinese players pretended to play harder after the referee's warning to improve performance after the first set, the numbers remained far off from Benford's distribution (χ^2=34.6, p=.001, ICC=.78).

Table 3: Distribution of FSD in anomalous match at 2012 London Olympic

Source	n	First digit									x^2	*P*	*ICC*
		1	2	3	4	5	6	7	8	9			
Benford	-	30.1	17.6	12.5	9.7	7.9	6.7	5.8	5.1	4.6	-	-	-
1st set	35	51.4	17.1	11.4	20.0	0	0	0	0	0	56.3	.001	.76
2nd set	32	37.5	6.2	15.6	0.0	6.3	9.4	12.5	3.1	9.4	34.6	.001	.78

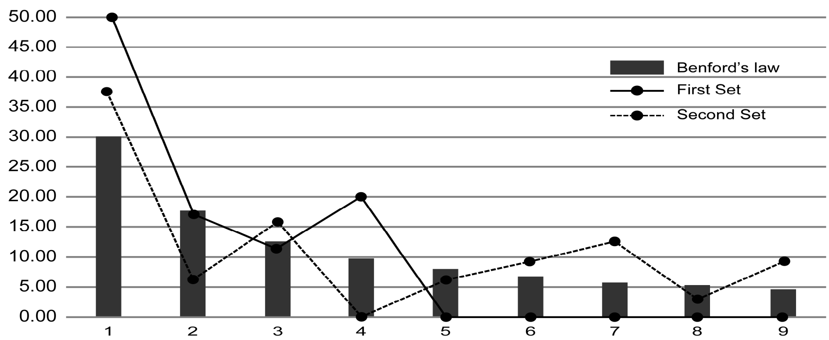

Figure 2: Distribution of FSD in anomalous match at 2012 London Olympic

Next case is 2008 All England Championship men's singles in which ranking 1st Chinese player who had already been qualified for Olympics deliberately lost against the same nationality player so as to participate together in the Olympics. The distribution of the number of rallies of the first (χ^2=24.5, p=.002, ICC=.86) and second (χ^2=19.1, p=.014, ICC=.87) sets was found to be significantly far off from Benford's expected distribution as in 2012 London Olympics case (Table 4 and Figure 3).

Table 4: Distribution of FSD in anomalous match at 2008 All England champion

Source	n	First digit 1	2	3	4	5	6	7	8	9	x^2	*P*	*ICC*
Benford	-	30.1	17.6	12.5	9.7	7.9	6.7	5.8	5.1	4.6	-	-	-
1st set	42	26.2	16.7	14.3	7.1	14.3	0.0	2.4	9.5	9.5	24.5	0.001	0.86
2nd set	48	37.5	12.5	6.3	6.3	14.6	2.1	6.3	8.3	6.3	19.1	0.014	0.87

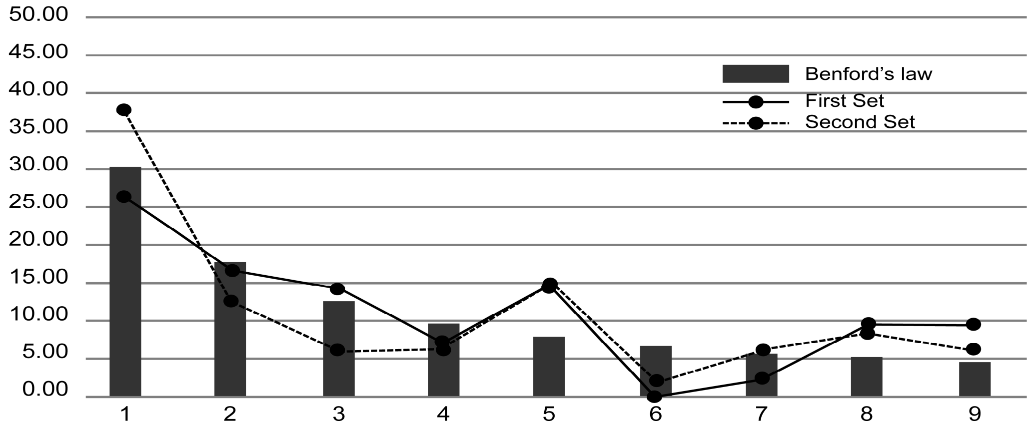

Figure 3: Distribution of FSD in anomalous match at 2008 All England Champion

4. Discussion

Benford's law has been recognized as a means to determine abnormal occurrences in numbers of real life (Nigrini, 1999). It is already used as a basic step to screen abnormality in numbers from financial statements (Rauch, et. al., 2011) and recently, to identify abnormal human behavior. For example, Golbeck (2015) attempted social media behavioral analysis by applying Benford's law to the number of followers in social networks such as Facebook and Twitter for analyzing human behavior, and sought to increase the validity of responses to sensitive questions in surveys by applying Benford's law to randomized responses.

In addition, Kreuzer, Jordan, Antkowiak, Drexler, Kochs, & Schneider (2014) have used Benford's law in detecting brain diseases through the analysis of the number of signals collected from brain waves. The present study applied Benford's law to the number of badminton rallies with an interest in whether sports data record conforms to Benford's law. As a result, it was found that the number of badminton rallies follows Benford's law. Regardless of the events (men's singles, women's singles, men's doubles, women's doubles, and mixed doubles), the number of badminton rallies is in conformity with Benford's law. This study has found that the number of badminton rallies generated by players competing against each other to win a game is spontaneous. This fact means that players competing for victory cannot deliberately control the number of rallies. This study can also be a basis for identifying anomalous badminton games that do not involve fierce competitions. The number of badminton rallies in two well-known anomalous cases was applied to Benford's law, and an interesting fact that those two anomalous cases are not in conformity with Benford's law was found in this study.

The main finding of the study is that the fixed matches between the Korean and Chinese players (intentional defeat) during the 2012 London Olympics, and between the Chinese and Chinese players during the 2008 All England Championship were far off from Benford's law. During the 2012 London Olympics and the 2008 All England Championship, some badminton players attempted to lose intentionally and did not play hard (Chappelet, 2015). The finding that the fixed match does not conform to Benford's law implies that the badminton rally data may not be generated randomly when artificial behavior is present in sports competition. Although evidence that is more empirical is needed as this finding is based on only two cases, the study has discovered the possibility of identifying sports match fixing in data.

What does it mean when data is generated randomly in sports competition? When two players compete fiercely to win a game, randomness of data on match records may be obtained. For example, if the difference in such players' skills is very large, the superior player can perform the game easily, and thus, sports data may not be random. Furthermore, the data collected from the case in which a player engages in a game with an intention of match-fixing may not have randomness as well. Future studies should analyze the characteristics of numbers that are generated randomly by selecting more cases of match-fixing and reflecting various variables such as skill differences between the players and/or the external environment.

Today, match-fixing has emerged as a social problem that threatens the presence of modern sports itself. Match-fixing is a universal phenomenon and a serious issue that affects the development of sports. It is neither confined to specific sports such as soccer (Abbott & Sheehan, 2014), figure skating (Clarey, 2002), cricket (Carpenter, 2012), basketball (Cohen, 2008), and motor racing (Cary, 2010), nor is it confined to specific nations. Former president of the International Olympic Committee (IOC) Jacques Rogge defined match-fixing as a "cancer," adding, "Doping affects one individual athlete, but the impact of match-fixing affects the whole competition. It is much bigger." (Chappelet, 2015, p. 13). In response, the IOC and the Fédération International de Football Association (FIFA) are announcing plans to take strong actions to address match-fixing.

After doping, match-fixing has been globally considered as a new scourge of sports, and sports organizations have started to fight against match-fixing as they did against doping problems a few decades ago (Chappelet, 2015). To prevent match-fixing, the IOC has been presenting various solutions such as the establishment of a department in charge, preventive education, and a system for whistle blowing (IOC, 2015). Further, the IOC focused on implementing the Early Warning System (Carpenter, 2012; IOC, 2015), which monitors the occurrence of match-fixing and gives out early warnings to the stakeholders of the game as like players, gamblers, or bookies.

This preliminary study would be a meaningful contribution to the field in that it has found the possibilities of monitoring suspected match-fixing cases through a statistical approach in sports data. As match-fixing is done secretively by players, coaches, and executives, detecting anomalies of players involved in match manipulation is considered a difficult job. Further, with the method proposed in this study, it may be impossible to prove whether players make mistakes with an intention.

However, the fact that the games in which players deliberately underperform do not conform to Benford's law while other games do can be a theoretical basis of identifying the anomalies that players make during the games. It is reported that 80% of the world's match-fixing scandals occur in soccer and basketball (Vodde, 2013), which offer a variety of records for not only the team and but also the individual players.

For instance, whether the records such as the distance covered or time taken for soccer players to sprint during the game, the number of touches to the ball while dribbling, and the number of passes of the ball during basketball games follow Benford's law would be an interesting topic for future studies. By considering discerning the elusive incidents of match-fixing carried out between sports players, the present study can lead to various follow-up studies to find evidence of match-fixing that appears in the data.

In conclusion, Benford's law is a useful means to identify abnormal numbers in various areas, and the present study found that the number of badminton rallies also follow the law. By using this law, the study confirmed that the possibility of identifying abnormality of badminton rally number generated by activities such as match-fixing.

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